Radioactive Decay and Modeling with Exponential Functions:

A great STEM combo

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Overview:

1. STEM at Groton School

2. Decay simulation

3. Graph of Data

4. Mathematical Treatment
Why STEM at Groton

- Current space insufficient
- Internal curriculum assessment
- New course: integrated science and math
- Raise the cache of the study of science and math
- New facility planned
History—STEM at Groton School

In 2010 an integrated STEM course for a select group of 9th graders beginning the 2011 school year was created.

In 2012 school year, the second-year course for 10th graders was initiated.

2013 is the third year of STEM1 and second year of STEM2.
9th Grade Curriculum

- **Science**—
  - chemical reactions, carbon cycle and climate change

- **Technology**—
  - calculators, Excel, Sketchpad and programming

- **Engineering**—
  - still developing

- **Mathematics**—
  - Geometry plus selected topics
10th Grade Curriculum

- **Science**—
  - atoms, crystals, polymers and cells

- **Technology**—
  - calculators, Excel, Sketchpad and programming

- **Engineering**—
  - rocket stove and parabolic solar collector

- **Mathematics**—
  - Algebra 2 plus selected topics
How do you introduce Nuclear Chemistry?

- Introduce Isotopes, Atomic Mass, and Symbols?
- Introduce Periodic Table and Average Atomic Mass?
- Introduce Radiation and Radioactivity?
How old is???
Learn what you need to know when you need to know it.

What evidence do you have that you are trying to explain?

Give them an experience that they need to explain that goes beyond the previous model.
Exponential Inquiry

Handout: “Headsium” Activity

- 50 “coinium” used instead of 100.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Nh--G1</th>
<th>Nh--G2</th>
<th>Nh--G3</th>
<th>Nh--G4</th>
<th>Nh--Total</th>
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<td>9</td>
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<td>6.25</td>
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</table>
## Data & Graph: Nh

### Time (s) & Nh Values

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>G1</th>
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<th>G3</th>
<th>G4</th>
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**Class Data: Headsium Left as time elapses**

**Class Average of Headsium Data as time elapses**
Direct Variation
- $y = mx + b$

Indirect Variation
- $y = a(1/x) + c$
Math from Unit 1

- **Function, \( f(x) \)**
  - \( f(x) = mx + b \)
  - \( f(x) = 2x + 3 \)

- **Inverse function, \( f^{-1}(x) \), for linear equations**
  - \( y = 2x + 3 \)
  - \( x = 2y + 3 \)
  - \( y = \frac{1}{2}(x - 3) \)
  - \( f^{-1}(x) = (1/2)*(x-3) \)
Other Math Connections:

- Determining differences in Excel.
- Observe that the “difference” is not constant.
Analysis–Growth:

<table>
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<tr>
<th>Nt--G1</th>
<th>Nt--G2</th>
<th>Nt--G3</th>
<th>Nt--G4</th>
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<td>95</td>
<td>99</td>
<td>97.25</td>
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</table>

"Tailsium"--Nt formed during reaction

\[ y = 0.4017x + 21.512 \]

\[ R^2 = 0.8358 \]
Data Analysis:

- Fit data using Excel to linear trend-line

- Q: What does the line in the data mean?
  - Minimize difference between the points and the average.

- Q: What does the $R^2$ mean?
  - Difference squared then averaged
Traditional Half-life Treatment:

- \( \frac{A_t}{A_o} = (1/2)^n \)
  - n = # half-lives
  - n = \( t/t_{1/2} = \frac{\text{time elapsed}}{\text{time of half-life}} \)

- 1 ➔ \( (1/2)^1 \) ➔ \( (1/2)^2 \) ➔ \( (1/2)^3 \) ➔ \( (1/2)^4 \) ➔ \( (1/2)^5 \)

- 1 ➔ \( \frac{1}{2} \) ➔ \( \frac{1}{4} \) ➔ \( \frac{1}{8} \) ➔ \( \frac{1}{16} \) ➔ \( \frac{1}{32} \)

- 100% ➔ 50% ➔ 25% ➔ 12.5% ➔ 6.25% ➔ 3.125%
Half of the radioactive atoms decay each half-life.

Radioactive decay

- Percentage of original sample
- Time (half-lives)
Without Logarithms:

- Graphical estimate of half-life

- Calculate amount for any number of half-lives
  - If gallium–68 has a half-life of 68.3 minutes, how much of a 23.5 mg sample is left after two half-lives?

- Calculation of number of half-lives limited to whole numbers
  - If 0.734 mg of sample is left, how many half-lives has passed?
Radon–222 is a Gas that is suspected of causing lung cancer as it leaks into houses. It is produced by Uranium decay. Assuming no loss or gain from leakage, if there is 1024 g of Rn–222 in the house today, how much will there be in 5.4 Weeks? (Rn–222 Half–Life Is 3.8 Days.)
Sample Problem Con’t:

How much will there be in 5.4 Weeks? (Rn–222 Half-Life Is 3.8 Days.), Continued

\[
5.4 \text{ weeks} \times 7 \frac{\text{days}}{\text{week}} = 37.8 \approx 38 \text{ days}
\]

<table>
<thead>
<tr>
<th>Amount of Rn-222</th>
<th>Number of Half-lives</th>
<th>Time (days)</th>
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<td>512 g</td>
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<td>3.8</td>
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<td>256 g</td>
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<td>32 g</td>
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<table>
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<tr>
<th>Amount of Rn-222</th>
<th>Number of Half-lives</th>
<th>Time (days)</th>
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<td>16 g</td>
<td>6</td>
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<td>8 g</td>
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<td>4 g</td>
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<td>2 g</td>
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<td>34.2</td>
</tr>
<tr>
<td>1 g</td>
<td>10</td>
<td>38</td>
</tr>
</tbody>
</table>
Solution using Exponentials:

- $5.4 \text{ weeks} \times 7 \left( \frac{\text{days}}{\text{week}} \right) = 37.8 \approx 38 \text{ days}$

- $n = 38 \text{ days} \times \left( \frac{\text{half-life}}{3.8 \text{ days}} \right) = 10 \text{ half-lives}$

- $A_t = A(10) = A_0 \left( \frac{1}{2} \right)^{10} = 1024 \text{g} \times \left( \frac{1}{2} \right)^{10} = 1 \text{ gram}$
Exponential and Logarithmic functions are further developed by the math teacher.

Law of Logs and Exponentials can be used without being developed here.

Establish a connection between exponentials and logarithms with applications.
Math Connections

- Exponential Growth
  - Interest Rates
    - Compounded yearly, monthly, daily, etc.
  - Growth Rates
    - Populations, bacteria, etc.

- Exponential Decay
  - Drug dose
    - Time and concentration
  - Nuclear decay
Introduce Log and Anti-Log

- Exponential: \(10^3 = 1000\)
- Logarithm: \(\log_{10}(1000) = 3 = \log_{10}(10^3)\)
- Anti-log: 

- Exponential: \(10^2 = 100\)
- Logarithm: \(\log_{10}(100) = 2 = \log_{10}(10^2)\)
- Anti-log: 

- Exponential: \(10^1 = 10\)
- Logarithm: \(\log_{10}(10) = 1 = \log_{10}(10^1)\)
- Anti-log: 

- Exponential: \(10^0 = 1\)
- Logarithm: \(\log_{10}(1) = 0 = \log_{10}(10^0)\)
Log and Anti-Log Con’t:

- $2^3 = 8 \quad \log_2 (8) = 3 \quad = \log_2 (2^3)$

- $\frac{1}{32} = 2^{-5} \quad \log_2 \left(\frac{1}{32}\right) = -5 \quad = \log_2 (2^{-5})$

- $\frac{1}{27} = 3^{-3} \quad \log_3 \left(\frac{1}{27}\right) = -3 \quad = \log_3 (3^{-3})$
Laws of Exponents and Logs:

- \( U = x^a \)
- \( V = x^b \)
- \( U \cdot V = x^a \cdot x^b = x^{(a+b)} \)

- \( \log_x (U \cdot V) = \log_x (U) + \log_x (V) \)
  \[ = \log_x (a) + \log_x (b) \]

- \( \log (x^n) = n \cdot \log (x) \)
With Exponentials:

- \( f(x) = C \cdot a^x \)
  - \( f(x) = \) amount at time \( x \)
  - \( C = f(0) = \) initial amount
  - \( a = \) decay factor
  - \( x = \) time periods elapsed (number of half-lives) = \( t / t_{1/2} \)

The half-life of carbon-14 is 5730 years. An ancient tree was discovered in the remains from a volcanic eruption. It was found to have 45.7% of the standard amount of carbon-14. When did the volcano erupt?
- Given: \( f(x) = 45.7\% \cdot C; a = \frac{1}{2}; t_{1/2} = 5730 \text{ yr.} \)
f(x) = C \cdot a^x
\frac{f(x)}{C} = \frac{0.457 \cdot C}{C} = \left(\frac{1}{2}\right)^x

0.457 = \left(\frac{1}{2}\right)^x

To solve for x need to use logarithms
\ln(0.457) = \ln(0.500)^x

Use Law of logs to solve for x...
\ln(0.457) = x \cdot \ln(0.500)
x = \ln(0.457)/\ln(0.500) = 1.130 = number of half-lives
t = x \cdot 5730 \text{ years} = 6473.375 \text{ years}
t \approx 6470 \text{ years}
Laws of Exponentials and Log allows you to show an alternative graphical relationship for the data.

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Analysis:

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Natural Log of number of "Headsium"

\[ y = -0.0159x + 6.0447 \]
\[ R^2 = 0.9976 \]
Isotopes and Nuclide Notation

Types of Decays
- Particle Radiation
- Penetration
- damage from radiation

Transmutation Equations

Band of Stability
- Predicting type of decay

Nuclear Energy
- Fission Reactions
- Fusion Reactions
- Nuclear Waste & Yucca Mountain


Chapter 8: Exponential and Logarithmic Functions

http://www.willamette.edu/~cstarr/math139/ch8.pdf


Other Resources:

- National Safety Council: [www.nsc.org/issues/radisafe.htm](http://www.nsc.org/issues/radisafe.htm)
- [http://www.chemteam.info/Radioactivity/Radioactivity.html](http://www.chemteam.info/Radioactivity/Radioactivity.html)
- K. Gorski, Alpha/Beta Emission Simulation.doc
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- Funding from: Groton School
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- Contact info: skelly@groton.org